CHARACTERISTICS OF THE BEHAVIOR OF THE SMALL-SCALE TURBULENCE STRUCTURE IN THE BOUNDARY LAYER

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Results of theoretical and experimental studies of the structure of turbulence in the boundary layer are presented. It is shown that the spectral density of turbulence energy deviates systematically from Kolmogorov-Obukhov's law and a theoretical explanation of this deviation is given.

Recent experimental data show that Kolmogorov-Obukhov's theory of isotropic turbulence is not asymptotically accurate and the Kolmogorov constant is actually a variable depending on the intermittency factor [1]. In the wall flows the intermittency factor decreases as the distance from the wall increases. So, it should be expected that in the boundary layer the Kolmogorov constant will change accordingly. No results of more detailed experimental and theoretical studies of this phenomenon are known to the authors. Since this matter is of great practical importance, special experiments have been conducted to investigate the structure of wall turbulence in wind tunnels developed at the Warsaw Technical University and theoretical analysis of the phenomenon has been carried out.

Theoretical Analysis of the Process. As has been shown in [2, 3], many nonlinear effects of wall turbulence can be described by the model of the mixing length of the second-order accuracy. For the logarithmic part of the layer in which turbulent mixing is approximately symmetric, the second approximation of the model has the form [2-4]

$$U'^2 \simeq (LU'_y)^2 + r\,(L^2 U''_{yy})^2\,, \tag{1}$$

where r is the parameter of relaxation of disturbances.

Prandtl was the first to use this approximation to overcome the contradiction between the first-order model of turbulent mixing and experimental data on the velocity of fluctuation motion on the axis of the jet [4]. It also appeared useful in description of fluid flows with supercritical state parameters, in which the m-shaped velocity profile is observed [5]. In [2, 3] it is shown that equation (1) describes satisfactorily flows with injection and nonisothermal and gradient flows. According to [2-5], r is close to unity.

If Eq. (1) is taken as the definition of the main component of the wall turbulence energy, Kolmogorov-Obukhov's equation for the dissipation rate of the energy in the boundary layer should be slightly changed in accordance with this equation. To do this, we will consider the region of the wall flow in which generation of turbulence is equilibrated approximately by its dissipation. Assuming that fluctuation of the longitudinal component of the velocity U' makes the main contribution to the energy and taking into consideration that in the low-frequency region of the energy spectrum the mixing length L is approximately proportional to the dissipation length, we write

$$\nu_T\,(U'_y)^2 \simeq \alpha\nu_T\,(U'/L)^2\,, \tag{2}$$

where α is a constant of the two-parameter model of turbulence.

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From comparison of Eqs. (1) and (2) the conclusion follows:

$$\alpha^{-1} = 1 + r\,(LU''_{yy}/U'_y)^2 . \qquad (3)$$

The right-hand side of Eq. (2) is the rate of dissipation of the turbulence energy in the boundary layer:

$$\varepsilon \simeq \alpha \nu_T\,(U'/L)^2 . \qquad (4)$$

Assuming that the turbulent viscosity in the shear flow is related to the rate of dissipation in a universal way [6]:

$$\nu_T^3 \simeq L^4 \varepsilon , \qquad (5)$$

and that the rate of dissipation of turbulence is constant, we come to the conclusion that

$$U'^2 \simeq (1 + r\,(LU''_{yy}/U'_y)^2)\,(\varepsilon L)^{2/3} . \qquad (6)$$

The above equation can be considered as a refinement of Kolmogorov-Obukhov's hypothesis for the low-frequency region of the turbulent spectrum in the case of disturbed turbulence $|LU''_{yy}/U'_y| > 0$. In this expression r is a parameter unknown beforehand and associated with the whole set of assumptions made earlier. Equation (6) relates the Kolmogorov constant to the character of the velocity distribution in the wall flow. For boundary-layer flows the ratio U''_{yy}/U'_y is a function of the exponent in the relation $U \simeq U_0 y^m$. Therefore, the Kolmogorov constant can be expressed in the form

$$C \simeq C_0\,(1 + r\,((m-1)\,L/y)^2) .$$

This expression can be considered a constant only in the wall region, where the mixing length is a linear function of the coordinate $L \simeq y$ [7], and in the external part of the boundary layer, where the ratio L/δ tends to zero. In the case of infinitely large Reynolds numbers for the internal boundary of the layer we can write approximately $C \simeq C_0(1 + r)$. On the external boundary of the layer, where the ratio L/y is proportional to L/δ, $C \simeq C_0$. The limiting values of the constant C agree with experimental data of [1] ($C_{max} \simeq 2.25$, $C_{min} \simeq 0.8$) for $C_0 \simeq 0.8$ and $r \simeq 1.8$.

Using the approximate relation between the spectral function of turbulence and its energy [8]

$$E \simeq -\,\partial U'^2/\partial k$$

for the low-frequency part ($L \simeq 1/k$) of the inertia interval of the energy spectrum described by the equation

$$E \simeq A \varepsilon^{2/3} k^{-5/3} , \qquad (7)$$

we obtain the expression for the constant A

$$A \simeq 2C_0\,(1 + 4r\,(LU''_{yy}/U'_y)^2)/3 . \qquad (8)$$

With the account of the power velocity distribution in the boundary layer $U \sim y^m$, we have

$$A \simeq 2C_0\,(1 + 4r\,((m-1)\,L/y)^2)/3 .$$

The above equation shows that for the larger part of the boundary layer, where L cannot be considered as a linear fucntion of y, the coefficient A is a hyperbolic function of the distance from the wall.

The obtained relation is inconvenient for practical estimation of A since it requires determination of the characteristic linear scale of turbulence in the boundary layer with various kinds of disturbances. In order to solve the problem in a first approximation, use will be made of the relation between the mixing length L and the rms velocity fluctuation $\overline{(U'^2)}^{1/2}$: $L \simeq \overline{(U'^2)}^{1/2}/U'_y$. As a result, it is possible to obtain the relation between the constant A and the local turbulence level of the flow $\mathrm{Tu} = \overline{(U'^2)}^{1/2}/U$:

$$A \simeq 2C_0\,(1 + 4r\,((m-1)/m\,\mathrm{Tu})^2)/3\,. \tag{9}$$

The resultant expression indicates that A depends substantially on the character of the transverse velocity distribution (the parameter m) and the local turbulence level of the flow Tu. The parameters just enumerated affect A in the complex, which can be reasonably designated by $J = (m-1)\mathrm{Tu}/m$.

More rigorous analysis of the relation $A(J)$ based on the use of complete turbulence energy Eq. (1) for determination of L leads to the expression

$$A \simeq 2C_0\,(1 + ((1 + 4rJ)^{1/2} - 1)^2)/3\,. \tag{10}$$

Special experiments were conducted in wind tunnels of the Institute of Applied Mechanics, Warsaw Technical University, to verify the main conclusions following from the present analysis.

2. Experimental Setup and Measuring Methods. The experiments were carried out in two open wind tunnels. In the first tunnel, with a transverse dimension of 240 × 600 mm, the average flow velocity was about 30 m/sec. In the second tunnel, with a cross section of 300 × 300 mm, the flow velocity was 17 m/sec. The external turbulence level in the flows varied from fractions of a percent to ten percent. Measurements were conducted at different points across the boundary layer, in different cross-sections of the tunnel, and in different conditions of the flow, namely, on a smooth plate or on an aerofoil section. In the latter case both accelerated and decelerated wall flows could be studied.

A standard DISA 55 M 10 thermoanemometer sensor was used. The signal coming from the sensor to the computer was processed numerically. The thermoanemometer signal was passed through a DISA 55 M 26 filter and supplied to a 8-bit A/D converter. The level of signal amplification and compensation was controlled by a computer by means of two D/A converters. A special program controlling this process chose optimal conditions of the conversion. The maximum sample of the signal was restricted by the storage capacity of 30720 one-bit signals. The maximum frequency of the sample was 40 kHz. More details about the signal converter can be found in [9]. After the A/D converter the output data of the anemometer were supplied not to an analog DISA linearizer but to the input of the computer, which allowed the velocity to be calculated as a linear function of the signal level. Two linearization methods were used here, namely, King's curve (the DISA method) and the fourth-power polynomial (the TSI method). Both methods gave similar results. Errors caused by the use of the numerical scheme of linearization of the signal appeared smaller that those arising with the use of the analog converter. The time dependence of the instantaneous velocity was recorded on a disk.

Subsequently, with a special data processing program the average and rms gas velocities we determined the autocorrelation functions, and spectral analysis of the signal was carried out by a fast Fourier transform. Moreover, the relation $\tau \simeq (2\overline{U'^2})^{1/2}/(\overline{(\partial U/\partial t)^2})^{1/2}$ was used to determine the time microscale of turbulence.

The dissipation rate of turbulence was found either from the relation

$$\varepsilon \simeq 15\nu\,\overline{((\partial U/\partial t)/U)^2}\,,$$

or with the use of the dissipative integral of the spectral function

$$\varepsilon_f \simeq 15\nu\,\overline{(\partial U'/\partial x)^2}\,, \quad \overline{(\partial U'/\partial x)^2} = \int F(k)\,k^2 dk\,, \quad k = 2\pi f/U\,.$$

The dissipation rates determined with the two methods differ from each other by approximately a factor of two: $\varepsilon_f/\varepsilon \simeq 1.5-2.5$. No steady correlation has been found between this ratio and any calculated or measured parameter.

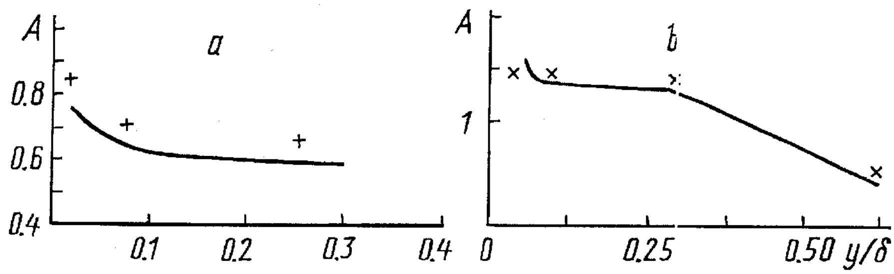

Fig. 1. Distribution of the coefficient A in the boundary layer on the aerofoil section (dots, experiment; curves calculated from Eq. (10)): a) maximum turbulence level Tu = 0.08%; b) Tu = 0.16%.

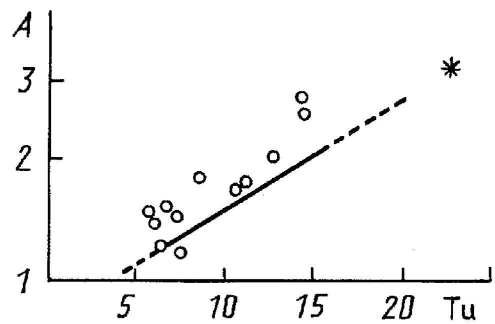

Fig. 2. Plot of the coefficient A versus the local turbulence level in the boundary layer on a plate (dots, experiment; curve calculated from Eq. (10)).

The results of measurements of the spectral density of turbulent fluctuations in the boundary layer were used to determine the parameter A in Eq. (7). The order of determination of A was as follows. In the plot of the spectral density of the energy versus the wave number plotted in logarithmic coordinates, the cross-section of cascade energy transfer over the spectrum was found with the aid of a straight line with a slope of $-5/3$. In this region the wave number was associated with the spectral density of the energy. On this basis the product $\varepsilon^{2/3}A$ was determined. Then, with the use of the precalculated rate of dissipation the constant A was calculated. The values of A in the boundary layer under different conditions of the flow surface determined in this way are shown in Figs. 1 and 2.

Figure 1 shows results of measuring the constant A in the boundary layer on an aerofoil section obtained in the first of the wind tunnels described here. The main difference in the experimental conditions considered can be explained by an approximately two-fold change in the maximum turbulence level ($Tu_a \simeq 0.08$ and $Tu_b \simeq 0.16$).

The curve shown in Fig. 2 was obtained in experiments on a smooth plate in the second of the wind tunnels described. The distribution of the average flow velocity, rms fluctuations of the velocity, time microscale of turbulence, and its rate of dissipation in the layer corresponding to these experiments are shown in Fig. 3.

3. Results of Comparison of Experimental Data with the Theoretical Model of the Phenomenon. The maximum theoretical value of A in the boundary layer occurs near the wall, where the mixing length can be assumed to be a linear function of the coordinate. According to Eq. (9), we obtain $A \simeq 3.3$ (this value is marked by an asterisk in Fig. 2). In the other part of the boundary layer the mixing length either grows slower or remains constant. Since the local turbulence level was measured in the present experiments and it was possible to approximate the average flow velocity by a power function, the solid line in Fig. 2 shows a theoretical curve plotted from Eq. (10).

Comparison of experimental data and theoretical curves plotted from Eq. (10) for the corresponding experiment conditions (see Fig. 1) shows their fair agreement.

If experimental data on A obtained in different conditions of the flow and at different points in the boundary layer are plotted in the coordinates A/A_0, J^2, it is possible to obtain the universal dependence of the Kolmogorov constant on the nonequilibrium level of the wall turbulence defined by J (Fig. 4). In this figure the solid line is the

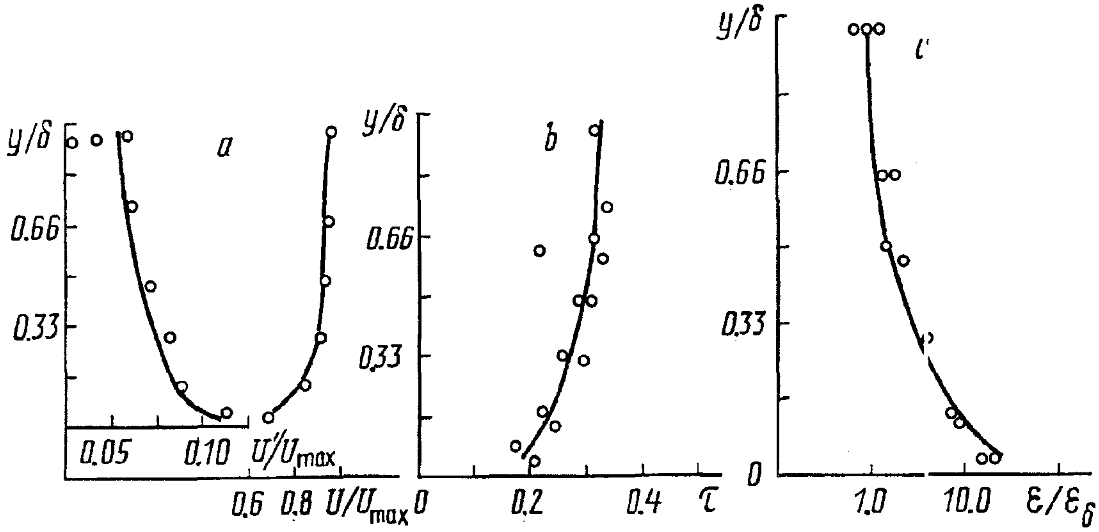

Fig. 3. Distribution of the main physical parameters of the boundary layer flow on a plate: a) velocity and its rms fluctuations; b) microscale of turbulence; c) relative dissipation rate of energy. τ, msec.

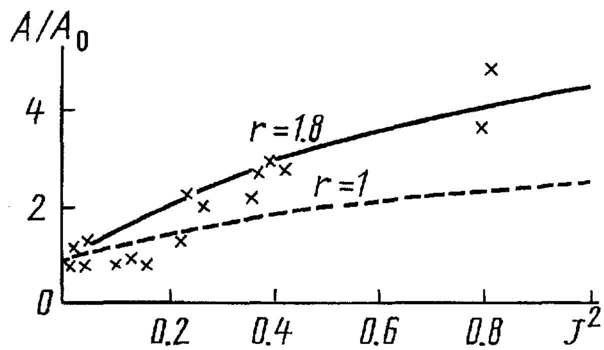

Fig. 4. Plot of A/A_0 versus the nonequilibrium parameter J^2 (dots, experiment; curves calculated from Eq. (10)).

curve plotted from Eq. (10) in terms of the constants $C_0 = 0.8$ and $r = 1.8$ determined independently in experiments in [1]. In these coordinates it is clearly seen that the ratio A/A_0 deviates systematically from unity. Substantial scatter of the experimental data observed in the figure can be explained by a rough approximation of the velocity profile by one exponent m throughout the whole boundary layer.

Conclusions. From the present study it can be concluded that in the region of cascade energy transfer the spectral energy density depends substantially on the local inhomogeneity of the averaged flow characterized by the ratio of the second and first derivatives of the averaged velocity over the transverse coordinate. In the range of the parameters of the main flow and the boundary layer that characterized the present experiments (the turbulence level Tu reached 30%), at negative and positive pressure gradients up to the separation point at different points inside the boundary layer, a common dependence of the coefficient A on the local inhomogeneity of the averaged flow was found. The dependence of the rate of dissipation of the turbulence energy on the local inhomogeneity of the flow expressed by Eqs. (3) and (4) should be taken into consideration in construction of semiempirical models of turbulent transfer.

NOTATION

U, average flow velocity; $U_y^{'}$, $U_{yy}^{''}$, first and second derivatives of the velocity over the transverse coordinate; $U^{'}$, rms velocity fluctuations; L, mixing length or dissipation length; r, parameter of the model; ν, ν_t, molecular and turbulent viscosities; ε, rate of dissipation of turbulence energy; ε_δ, rate of dissipation on the boundary of the layer; C, Kolmogorov constant; k, wave number; E, spectral function of turbulence; A, constant; J, dimensionless parameter; Tu, local turbulence level of the flow; m, exponent of the velocity function; τ, microscale of turbulence.

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